O K L A H O M A S T A T E U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING

ECEN 5513
Stochastic Systems
Fall 2007
Midterm Exam #2

PLEASE DO ALL FIVE PROBLEMS

Name : _____

E-Mail Address:_____

Problem 1:

A random variable Θ is uniformly distributed on the interval (θ_1, θ_2) where θ_1 and θ_2 are real and satisfy $0 \le \theta_1 < \theta_2 < \pi$. Find and sketch the probability density function of the transformed random variable $Y = \cos(\Theta)$.

Problem 2:
In a computer simulation, it is desired to transform numbers that are values of a random variable uniformly distributed on $(0,1)$ to numbers that are values of a Cauchy random variable with distribution function

$$F_X(x) = \frac{1}{2} + \frac{1}{\pi}\tan^{-1}\left(\frac{x}{b}\right).$$

Find the required transformation.

Problem 3:

The *non-negative* random variables *X* and *Y* are statistically independent with exponential densities

$f_X(x) = \alpha e^{-\alpha x} u(x)$, and

$f_Y(y) = \beta e^{-\beta y} u(y)$.

Find the probability density function of the random variable $W = X / Y$.

Problem 4:

Prove the Schwarz's inequality for random variables X ad Y:

$$\left[E(XY)\right]^2 \le E(X^2)E(Y^2).$$

Hint: Homework 7, Problem 6.

Problem 5:

Random variables X_1 and X_2 having the joint density

$$f_{X_1,X_2}(x_1,x_2) = \tfrac{3}{8}u(x_1-2)u(x_2-1)x_1x_2^2\exp(4-2x_1x_2)$$

undergo a transformation

$$[T] = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

to generate new random variables Y_1 and Y_2. Find the joint density of Y_1 and Y_2, $f_{Y_1Y_2}(y_1,y_2)$.